Exercise 9

(a) Use a graph to find a number δ such that

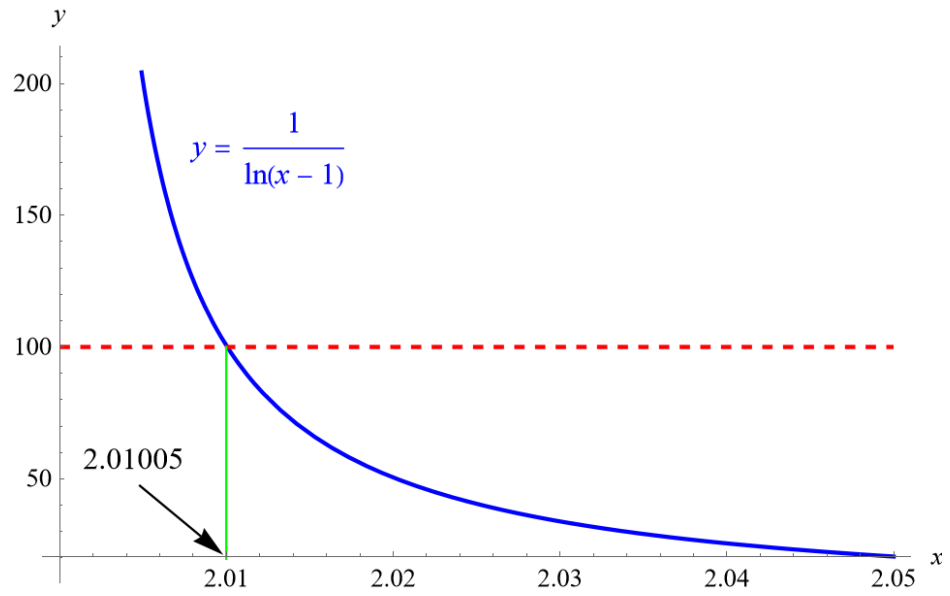
$$\text{if} \qquad 2 < x < 2 + \delta \qquad \text{then} \qquad \frac{1}{\ln(x-1)} > 100$$

(b) What limit does part (a) suggest is true?

Solution

Part (a)

Below is a graph of $1/[\ln(x-1)]$ versus x.



For $1/[\ln(x-1)]$ to be greater than 100, δ has to be less than about 0.01005.

Part (b)

$2 < x < 2 + \delta$ indicates that x goes to 2 from the right, and $1/[\ln(x-1)] > 100$ indicates that the function is $1/[\ln(x-1)]$ and that the limit is infinity (Definition 6 on page 112).

$$\lim_{x \to 2^+} \frac{1}{\ln(x-1)} = \infty$$

To verify this limit, plug in 2^+ for x.

$$\lim_{x \to 2^+} \frac{1}{\ln(x-1)} = \frac{1}{\ln(2^+ - 1)} = \frac{1}{\ln 1^+} = \frac{1}{0^+} = \infty$$